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Extended BRS invariance and OSp(4/2) supersymmetry

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Abstract. A superfield action is proposed within an OSp(4/2) framework whose component form reproduces the covariant ξ-gauge Yang–Mills action, but with modified ghost-compensating terms. (The case $\xi = 0$ reduces to the usual Landau gauge.) 'Supertranslations' give rise to extended BRS transformations, and lead to constraints amongst the renormalisation constants. In addition, the system admits 'super-Lorentz' transformations, which mix vector and ghost fields. For other field representations, the ghost structure suggested by the space–time supersymmetry OSp(4/G) is also exhibited. This simplifies the rules for counting ghosts and their own ghosts.

1. Introduction and main results

It has long been recognised that the so-called BRS symmetry (Becchi *et al* 1975, 1976) of the gauge-fixing plus ghost-compensating Lagrangian in Yang–Mills theories (Feynman 1963, De Witt 1965, Faddeev and Popov 1967) has powerful implications for their quantisation and renormalisation. Subsequent investigations have revealed that an 'extended' BRS set can be constructed (Curci and Ferrari 1976, Ojima 1980), involving a two-parameter 'BRS group' where the roles of 'ghost' and 'antighost' can essentially be interchanged. Following earlier work on the unextended case (Ferrara *et al* 1977, Fujikawa 1978), Bonora and Tonin (1981) in an important recent paper have presented a concise derivation of the extended BRS transformations from a manifest superfield formalism, in which the BRS group consists of supertranslations in the a-number superspace coordinates.

Several arguments can be adduced leading to the possible existence of a supersymmetric BRS formalism. Firstly, the fact that the gauge potential $A^a_\mu(x)$ is accompanied by a-number 'ghost' fields, denoted here by $\omega^a(x)$ and $\bar{\omega}^a(x)$, all transforming in the adjoint representation of the gauge group, is highly reminiscent of Fermi–Bose supersymmetry, where gauge supermultiplets, including vector bosons and Majorana fermions, necessarily fall into the adjoint representation (for a review, see Fayet and Ferrara (1977)). Secondly, there is a natural 'auxiliary field' formalism for gauge fixing, whereby the usual covariant term $-(\partial^\mu A_\mu)^2/2\xi$ is replaced by

$$(\partial^\mu A_\mu)B + \tfrac{1}{2}\xi B^2,$$

where $B(x)$ is an auxiliary field of dimension two. $B(x)$ here plays a role similar to that of the auxiliary field in Fermi–Bose sypersymmetry. Finally, it is to be expected that a consistent supersymmetric derivation would provide a natural rationale for the existence of the two-parameter extended BRS group, in which the ghost and antighost fields ω and $\bar{\omega}$ are placed on an equal footing.

It is found (Bonora and Tonin 1981) that the appropriate supersymmetry involves a six-dimensional superspace with coordinates $(x_\mu, \theta, \bar{\theta})$. Making superfield expansions on the a-number coordinates θ and $\bar{\theta}$, Bonora and Tonin (1981) impose constraints on the 'supercurvature' and obtain sufficient conditions that supertranslations on θ and $\bar{\theta}$ correspond to extended BRS transformations, and the most general supersymmetric Lagrangian density is a two-gauge parameter generalisation of the usual Yang–Mills ghost Lagrangian. In a related paper, Bonora *et al* (1980) have justified their derivation in terms of a geometrical construction, enabling an interpretation to be given to terms such as 'connection' and 'curvature', which were undefined in the original work.

It is the purpose of the present paper to present an alternative formulation of BRS supersymmetry. We go beyond the work of Bonora and Tonin (1981) and Bonora *et al* (1980) in the sense that their group of 'supertranslations' in $(\theta, \bar{\theta})$ space is here enlarged to include transformations mixing x_μ and $(\theta, \bar{\theta})$. The appropriate supersymmetry is a real form of the inhomogeneous OSp(4/2) supergroup (Dondi and Jarvis 1979), consisting of supertranslations, Lorentz transformations, symplectic transformations in $(\theta, \bar{\theta})$ space, as well as 'supertranslations' and 'super-Lorentz' transformations (§ 2). With the help of this space–time supersymmetry, the definitions and transformation properties of the gauge potential and field strength follow naturally (§ 3), without further geometrical constructions.

The formalism of a gauge theory over six-dimensional superspace having been introduced, attention is restricted to a special class of potentials (beyond what gauge freedom alone would allow). An action for pure Yang–Mills theory, with gauge-fixing and ghost-compensating terms, is derived in § 3 as the component form of an appropriate superfield action, wherein the gauge potential belongs to the special class. The Lagrangian, equation (15), is a particular case of that of Bonora and Tonin (1981), wherein the two independent gauge parameters are made equal. There is a quartic ghost self-coupling, and the vector–ghost coupling attains a symmetrical form reminiscent of charged scalar electrodynamics. These additional couplings fade out as $\xi \to 0$, and in this limit with $\partial^\mu A_\mu = 0$, the model reduces to the conventional Landau gauge.

Supertranslations give rise to extended BRS transformations amongst the component fields (as in Bonora and Tonin 1981). These are exposed in § 4, and used to derive generating functional equations. The resulting BRS identities (for the effective action) imply renormalisability of the model. Relations between the renormalisation constants are exhibited, and verified to one-loop order. The only difference from the usual Yang–Mills case is in the fact that the longitudinal part of the vector boson propagator (and hence the gauge parameter ξ) receives a separate renormalisation. The transverse part continues to obey the usual Slavnov–Taylor identity. Moreover, the symmetry of the model ensures that all counterterms already have counterparts in the bare Lagrangian. In particular, renormalisation respects the symmetrical form of the vector–ghost interaction.

The space–time supersymmetry we propose also admits super-Lorentz transformations, which mix the coordinates x_μ and $(\theta, \bar{\theta})$. In contrast to supertranslations, the class of gauge potentials of interest is not super-Lorentz invariant. Nonetheless one can implement a corresponding set of vector–ghost mixing transformations (§ 5). The resulting identities for the effective action are also presented and checked.

Concluding remarks are made in § 6, together with a comparison with other approaches. Finally, the ghost structure suggested by the OSp(4/2) supersymmetry is given for other field representations, including the totally symmetrical and mixed-symmetry rank-three gauge fields. This represents a tidy and effective way of counting ghosts and superghosts for any chosen field representation.

2. Space–time supersymmetry

The space–time supersymmetry which we impose is a real form of the six-dimensional inhomogeneous orthosymplectic supergroup $\mathrm{OSp}(4/2) \wedge T_{4/2}$, the group of all superlinear transformations preserving the distance (Dondi and Jarvis 1979)

$$(X-Y)^2 \equiv (X-Y)_u g^{uv}(X-Y)_v$$

between points in superspace. Taking $X_u = (x_\mu, \theta_\alpha)$, where $\mu = 0, 1, 2, 3$ and $\alpha = 1, 2$, we have

$$X_u g^{uv} X_v = x^\mu x_\mu + \theta^\alpha \theta_\alpha. \tag{1}$$

Here the orthosymplectic metric is

$$g^{uv} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & \varepsilon^{\alpha\beta} \end{pmatrix}$$

where $\eta^{\mu\nu}$ is the usual diagonal Lorentz metric, and $\varepsilon^{\alpha\beta}$ is the 2×2 antisymmetric matrix,

$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The real form is determined by that of the underlying Lie group (Parker 1980), which we take to be simply $\mathrm{O}(3, 1)\times \mathrm{Sp}(2, R)$, namely the Lorentz group together with 2×2 real symplectic transformations. The group $\mathrm{Sp}(2, R)$ is more familiar as $\mathrm{SL}(2, R)$, locally isomorphic to the three-dimensional Lorentz group $\mathrm{SO}(2, 1)$. The θ_α thus transform as a spinor representation of this group.

Since the group matrices are real, one can in principle take θ_α to be real. However, to ensure the reality of the bilinear form (1), with the usual properties of complex conjugation for a-numbers, a different assignment must be made. The choice adopted in the usual BRS formalism (Bonora and Tonin 1981, Kugo and Ojima 1979),

$$\theta_1^* = \theta_1, \qquad \theta_2^* = -\theta_2,$$

is one possibility. However, the $\mathrm{Sp}(2, R)$ symmetry ensures that for the present case, the choice $\theta_2 = \theta_1^*$ is an equally feasible alternative. This choice is adopted below, where we write $(\theta_1, \theta_2) = (\theta, \bar\theta)$ (Dondi and Jarvis 1979). Most of the formalism goes through for either choice; in the former case, it is more usual to write $\theta_2 = \mathrm{i}\bar\sigma$, $\theta_1 = \sigma$, and for the ghost fields $\bar\omega = \mathrm{i}\bar c(x)$, $\omega(x) = c(x)$, with σ, $\bar\sigma$, $c(x)$ and $\bar c(x)$ real (Bonora and Tonin 1981, Kugo and Ojima 1979).

In addition to the usual translations and Lorentz transformations of the Poincaré group, the space–time supergroup includes symplectic rotations on θ_α,

$$(x_\mu, \theta_\alpha) \to (x_\mu, \tau_\alpha^\beta \theta_\beta), \tag{2}$$

where $\tau_\alpha^\gamma \varepsilon^{\alpha\beta} \tau_\beta^\delta = \varepsilon^{\gamma\delta}$. There are also the supertranslations

$$(x_\mu, \theta_\alpha) \to (x_\mu, \theta_\alpha + \varepsilon_\alpha), \tag{3}$$

and the super-Lorentz transformations,

$$(x_\mu, \theta_\alpha) \to (x_\mu + \lambda^\beta_\mu \theta_\beta, \theta_\alpha - \lambda^\nu_\alpha x_\nu). \tag{4}$$

Clearly, for compatibility with the chosen reality conditions of the θ_α, we must have

$$\varepsilon_2 = \varepsilon_2^*, \qquad \lambda_2^\mu = \lambda_1^{\mu *}, \qquad \tau_2^2 = \tau_1^{1*}, \qquad \tau_2^1 = \tau_1^{2*}.$$

The superfield actions we shall construct require the group-invariant measure

$$d^6X = d^4x \, d\theta \, d\bar{\theta}.$$

That this is indeed invariant can be readily checked for supertranslations and symplectic rotations. For the super-Lorentz transformations, the same applies after use of the exponential formula to evaluate a superdeterminant.

3. Superfield formalism

In formulating a local gauge theory over superspace (Salam and Strathdee 1974), one encounters superfields

$$\Phi(x, \theta) = A(x) + \theta^\alpha \psi_\alpha(x) + \theta^\beta \theta_\beta B(x),$$

whose components in the Taylor expansion (a quadratic polynomial in θ) are ordinary fields. In particular, the gauge potential will be a superfield

$$\Phi_u(x, \theta) = (A_\mu(x), A_\alpha(x)) + (\text{higher-order terms}),$$

with $A_\mu(x)$ a c-number field, and $A_\alpha(x)$ an a-number field, transforming as the six-dimensional vector representation of OSp(4/2), and taking values in the Lie algebra of the gauge group (taken to be compact):

$$\Phi_u(x, \theta) = \Phi_u^a(x, \theta) T^a$$

where $[T^a, T^b] = \mathrm{i} f^{abc} T^c$, and f^{abc} are the totally antisymmetrical structure constants.

The gauge field strength $\Phi_{uv}(x, \theta)$ is a superfield transforming in the 17-dimensional graded-antisymmetrical tensor representation of OSp(4/2) (Dondi and Jarvis 1980, 1981). If we define the signature factor $[uv] = \pm 1$ such that $[\alpha\beta] = -1$ and $[\mu\nu] = [\mu\alpha] = +1$, we have, following the usual constructions (see, for example, Abers and Lee 1973),

$$\Phi_{uv} = \partial_u \Phi_v - [uv] \partial_v \Phi_u - \mathrm{i}e[\Phi_u, \Phi_v], \tag{5}$$

where e is the gauge coupling constant. Thus

$$\Phi_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu - \mathrm{i}e[\Phi_\mu, \Phi_\nu],$$

$$\Phi_{\mu\alpha} = \partial_\mu \Phi_\alpha - \partial_\alpha \Phi_\mu - \mathrm{i}e[\Phi_\mu, \Phi_\alpha],$$

$$\Phi_{\alpha\beta} = \partial_\alpha \Phi_\beta + \partial_\beta \Phi_\alpha - \mathrm{i}e[\Phi_\alpha, \Phi_\beta]_+,$$

with

$$\Phi_{\mu\nu} = -\Phi_{\nu\mu}, \qquad \Phi_{\mu\alpha} = -\Phi_{\alpha\mu}, \qquad \Phi_{\alpha\beta} = \Phi_{\beta\alpha}.$$

Gauge transformations of Φ_u and Φ_{uv} under (x, θ)-dependent elements $U(x, \theta)$ of the gauge group are given as usual by

$$\Phi'_u(x, \theta) = U^{-1}\Phi_u(x, \theta)U - (\mathrm{i}/e)(\partial_u U^{-1})U, \qquad \Phi'_{uv}(x, \theta) = U^{-1}\Phi_{uv}(x, \theta)U. \tag{6}$$

Any gauge transformation can be uniquely decomposed (Bonora *et al* 1980) as a product of a purely x-dependent piece $U_0(x)$ and an x- and θ-dependent piece $U_1(x, \theta)$:

$$U(x, \theta) = \{\exp[-ie\Lambda(x)]\}\{\exp[-ie(\theta^\alpha\omega_\alpha(x) - \theta^\beta\theta_\beta B(x))]\} \equiv U_0U_1. \tag{7}$$

At this stage we introduce the special class of gauge potential superfields $\Phi_u(x, \theta)$ which will be required in deriving the Yang–Mills action (Bonora and Tonin 1981). Namely, we restrict attention henceforth to those potentials which are related by a gauge transformation to the special form in which the only non-vanishing component field is the ordinary four-vector potential $A_\mu(x)$. Without loss of generality, we may take the gauge transformation in question to be of the $U_1(x, \theta)$ type (since $U_0(x)$ does not mix components). For the special class we have

$$\Phi_u(x, \theta) = U_1^{-1}\begin{pmatrix} A_\mu(x) \\ 0 \end{pmatrix} U_1 - \frac{i}{e}(\partial_u U_1^{-1})U_1 \tag{8}$$

and

$$\Phi_{uv}(x, \theta) = U_1^{-1}\begin{pmatrix} F_{\mu\nu}(x) \\ 0 \\ 0 \end{pmatrix} U_1, \tag{9}$$

where $F_{\mu\nu}(x)$ is the usual Yang–Mills field strength.

It should be emphasised that this restriction is to be regarded as an additional assumption, beyond what is allowed from gauge freedom alone. For the present superfield formalism it corresponds in some sense to the usual procedure of gauge fixing and ghost compensation: this viewpoint is of course borne out by the final results.

The six-dimensional action is taken to be the sum of a gauge-independent piece and a gauge-dependent piece:

$$W(\Phi) = W_0(\Phi) + W_1(\Phi).$$

Observing that, if Φ_u belongs to the restricted class of potentials,

$$\Phi^{auv}\Phi^a_{vu} = (U_1^{-1}F_{\mu\nu}U_1)^a(U_1^{-1}F_{\nu\mu}U_1)^a = -F^{a\mu\nu}F^a_{\mu\nu}, \tag{10}$$

so that the usual Yang–Mills Lagrangian is formally invariant. The corresponding six-dimensional action is simply chosen to reflect this:

$$W_0 = \int X^2\, d^6X \cdot \tfrac{1}{4}\Phi^{auv}\Phi^a_{vu}. \tag{11}$$

Here the action of $d\theta\, d\bar{\theta}$ is to pick out the coefficient of $\theta^\alpha\theta_\alpha$, namely $-\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}$. Thus the supertranslation invariance of W_0 is also assured.

The choice of gauge-dependent action W_1 is not unique, and can only be guided by considerations of dimension, supersymmetry, and ultimately with a view to the required final form. The choice we adopt is

$$W_1 = \int d^6X \cdot 2\Phi^u\Phi_u/\xi, \tag{12}$$

where Φ_u is of the restricted form, and ξ is a real constant.

It remains to give $\Phi_u(x, \theta)$ and the component form of the action for the restricted class of potentials defined by (8) and (9). Expanding the exponential in (7), the finite group element $U_1(x, \theta)$ can be written

$$U_1(x, \theta) = 1 - \mathrm{i}e\theta^\beta\omega_\beta + \tfrac{1}{2}\mathrm{i}e\theta^\gamma\theta_\gamma(B - \tfrac{1}{2}\mathrm{i}e\omega^\delta\omega_\delta).$$

From (8), we have

$$\Phi_\mu = A_\mu + \theta^\beta D_\mu\omega_\beta - \tfrac{1}{2}\theta^\gamma\theta_\gamma[D_\mu B + \tfrac{1}{2}e(D_\mu\omega^\delta)\times\omega_\delta],$$
$$\Phi_\alpha = \omega_\alpha + \theta^\beta(B\varepsilon_{\beta\alpha} - \tfrac{1}{2}e\omega_\beta\times\omega_\alpha) - \tfrac{1}{2}\theta^\gamma\theta_\gamma[-eB\times\omega_\alpha + \tfrac{1}{6}e^2(\omega_\alpha\times\omega^\delta)\times\omega_\delta], \tag{13}$$

where D_μ is the covariant derivative,

$$D_\mu B = \partial_\mu B + eA_\mu \times B,$$

etc. Taking the $\theta^\alpha\theta_\alpha$ coefficient of $2\Phi^u\Phi_u/\xi$ gives with (10) the Minkowski-space form of the action†:

$$W = \int \mathrm{d}^4x\Big[-\tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \left(\frac{2}{\xi}\right)(\partial^\mu A_\mu\cdot B + B^2)$$
$$-\left(\frac{1}{\xi}\right)\partial^\mu\omega^\alpha\cdot D_\mu\omega_\alpha - \left(\frac{e^2}{12\xi}\right)\omega^\alpha\times\omega^\beta\cdot\omega_\alpha\times\omega_\beta\Big]. \tag{14}$$

In (13) and (14), the dot and cross products are generalisations of the usual vector notation:

$$\partial^\mu A_\mu\cdot B = \partial^\mu A^a_\mu B^a, \qquad (\omega_\alpha\times\omega_\beta)^c = f^{abc}\omega^a_\alpha\omega^b_\beta, \qquad \text{etc.}$$

With appropriate rescalings of B and ω_α, and expanding in terms of the spinor components ω and $\bar\omega$ the total Lagrangian becomes

$$\mathscr{L} = -\tfrac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + (\partial^\mu A_\mu\cdot B + \tfrac{1}{2}\xi B^2) - \partial^\mu\bar\omega\cdot\partial_\mu\omega - \tfrac{1}{2}eA^\mu\cdot\bar\omega\times\overleftrightarrow{\partial}_\mu\omega + \tfrac{1}{8}e^2\xi(\bar\omega\times\omega)^2. \tag{15}$$

Elimination of the auxiliary field B leads to the usual covariant gauge-fixing term $-(\partial^\mu A_\mu)^2/2\xi$. The present Lagrangian *differs from the conventional one* in the form of the vector–ghost coupling, and the quartic ghost self-coupling. It is a particular case of that of Bonora and Tonin (1981), in which the two independent gauge parameters they introduce are held equal. The fields

$$B_\pm = B \pm \tfrac{1}{2}e\bar\omega\times\omega \tag{16}$$

are the same as B, $\bar B$ in their notation, and the identification is completed by adopting the standard reality assignment $\bar\omega = \mathrm{i}\bar c$, $\omega = c$, with c, $\bar c$ real (see, for example, Kugo and Ojima (1979), and references therein). However, as emphasised above, the convention $\bar\omega = \omega^*$ is also feasible in the present formulation. The additional symmetry between ω and $\bar\omega$ forbids terms which would be formally non-Hermitian if $\bar\omega$ and ω are complex conjugates, and in particular leads the vector–ghost coupling to attain a form reminiscent of charged scalar electrodynamics. In the limit $\xi\to 0$, the additional terms fade out, and one is left with the conventional Landau gauge. The Feynman rules from (15) are the conventional ones for the vector field (see, for example, Abers and Lee 1973). Those for the ghost field are given in figure 1.

† It should be noted from (11) and (12) that the action is formally scale invariant at the classical level.

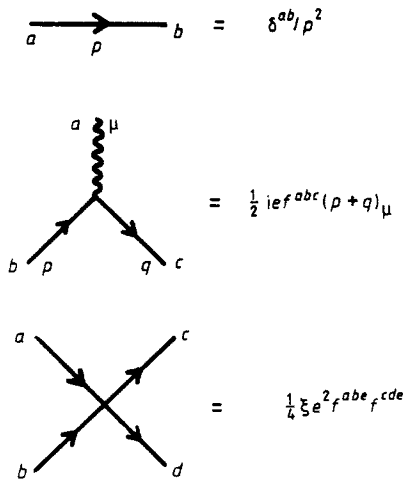

Figure 1. Feynman rules for ghost couplings.

4. Supertranslations and the effective action

The action (11) plus (12) is formally invariant under supertranslations

$$\delta\Phi_u(x,\theta)=\Phi_u(x,\theta+\varepsilon)-\Phi_u(x,\theta).$$

Moreover, it can be verified that the supertranslations respect the condition (8) defining the restricted class of gauge potentials. The component form of the supertranslations follows from (13). In terms of the spinor components ε and $\bar\varepsilon$, we have with (16)

$$\delta A_\mu=\bar\varepsilon D_\mu\omega-\varepsilon D_\mu\bar\omega,\qquad \delta\omega ff-\tfrac{1}{2}e\bar\varepsilon\omega\times\omega-\varepsilon B_-,\qquad \delta\bar\omega=-\bar\varepsilon B_++\tfrac{1}{2}e\varepsilon\bar\omega\times\bar\omega,$$
$$\delta B_+=0-e\varepsilon B_+\times\bar\omega,\qquad \delta B_-=e\bar\varepsilon B_-\times\omega+0. \tag{17}$$

These transformations thus provide an invariance of the total Lagrangian, as also follows explicitly from (15). This is the so-called extended BRS invariance. The symmetrical form with respect to $\bar\varepsilon$ and ε is in keeping with the present treatment of ω and $\bar\omega$ (in fact, the dual δ_ε transformations can be obtained from the $\delta_{\bar\varepsilon}$ transformations by Hermitian conjugation, if the convention $\bar\omega=\omega^*$ is adopted). The fact that the supertranslations anticommute is reflected in the so-called nilpotency of the BRS transformations, namely from (17)

$$\delta_{\bar\varepsilon}B_+=0=\delta_\varepsilon B_+\times\bar\omega,$$

and so on.

In following the implications of the BRS invariance for the quantised model and its renormalisation, it is convenient to eliminate B for $-(\partial^\mu A_\mu)/\xi$ (which is consistent with (17)). Also, we restrict attention only to the $\delta_{\bar\varepsilon}$-type BRS transformations. Finally, the nilpotency conditions allow the introduction of composite source terms in the Lagrangian:

$$\mathscr{L}_S=j^\mu\cdot A_\mu+\bar J\cdot\omega+\bar\omega\cdot J+\bar I^\mu\cdot D_\mu\omega-\tfrac{1}{2}e\bar{\bar I}\cdot\omega\times\omega-K\cdot B_+-e\bar K\cdot B_-\times\omega. \tag{18}$$

The equations of motion following from $\mathscr{L}-\mathscr{L}_S$ read

$$\partial^\mu D_\mu\omega+\tfrac{1}{2}e\xi(B_-\times\omega)=J+\tfrac{1}{2}eK\times\omega+\tfrac{1}{4}e^2\bar K\times(\omega\times\omega), \tag{19}$$

plus a similar (but more complicated) equation for $\bar{\omega}$, plus

$$D^{\mu}F_{\mu\nu}+\frac{1}{\xi}\partial^{\mu}(\partial^{\nu}A_{\nu})-\tfrac{1}{2}e\omega\times\overleftrightarrow{\partial}_{\nu}\omega=j_{\nu}-\bar{I}_{\nu}\times\omega-\frac{1}{\xi}\partial_{\nu}K+\frac{e}{\xi}\partial_{\nu}(\bar{K}\times\omega). \quad (20)$$

In (18) and (19), we have

$$B_{\pm}=-(\partial^{\mu}A_{\mu})/\xi\pm\tfrac{1}{2}e\bar{\omega}\times\omega \quad (21)$$

in place of (16).

Equations (19) and (20) impose conditions on the vacuum functional

$$Z(j^{\mu},J,\bar{J},\bar{I}^{\mu},\bar{I},K,\bar{K})=N\int \mathrm{d}[A,\omega,\bar{\omega}]\exp\Big(\mathrm{i}\int \mathrm{d}x\;(\mathscr{L}-\mathscr{L}_{\mathrm{S}})\Big);$$

for example

$$\partial^{\mu}\mathrm{i}\frac{\delta Z}{\delta\bar{I}^{\mu a}}=J+\frac{\xi\mathrm{i}}{2}\frac{\delta Z}{\delta\bar{K}^{a}}-\frac{ef^{abc}K^{b}\mathrm{i}}{2}\frac{\delta Z}{\delta\bar{J}^{c}}-\frac{ef^{abc}\bar{K}^{b}\mathrm{i}}{2}\frac{\delta Z}{\delta\bar{I}^{c}}. \quad (22)$$

Similarly from the supertranslations (17) one finds that

$$\int \mathrm{d}x\Big(j_{x}^{\mu a}\mathrm{i}\frac{\delta Z}{\delta\bar{I}_{x}^{\mu a}}-\bar{J}_{x}^{a}\mathrm{i}\frac{\delta Z}{\delta\bar{I}_{x}^{a}}-\mathrm{i}\frac{\delta Z}{\delta K_{x}^{a}}J_{x}^{a}\Big)=0 \quad (23)$$

from the variation of $\mathscr{L}_{\mathrm{S}}$. It is usual to pass to the connected vacuum functional W via $Z=\mathrm{e}^{\mathrm{i}W}$ and thence to the effective action

$$\Gamma(A,\omega,\bar{\omega},\bar{I}^{\mu},\bar{I},K,\bar{K})=W(j,J,\bar{J};\bar{I}^{\mu},\bar{I},K,\bar{K})+\int \mathrm{d}x\;(j^{\mu}A_{\mu}+\bar{J}\omega+\bar{\omega}J)$$

with the correspondences

$$\delta\Gamma/\delta A^{\mu}=j_{\mu}, \qquad \delta W/\delta j_{\mu}=-A^{\mu},$$

and so on. (22) and (23) become, for example,

$$\partial^{\mu}\frac{\delta\Gamma}{\delta\bar{I}^{\mu a}}=-\frac{\delta\Gamma}{\delta\bar{\omega}^{a}}+\frac{\xi}{2}\frac{\delta\Gamma}{\delta\bar{K}^{a}}+\tfrac{1}{2}ef^{abc}K^{b}\omega^{c}-\frac{ef^{abc}\bar{K}^{b}}{2}\frac{\delta\Gamma}{\delta\bar{I}^{c}} \quad (24)$$

and

$$\int \mathrm{d}x\Big(\frac{\delta\Gamma}{\delta A_{\mu x}^{a}}\frac{\delta\Gamma}{\delta\bar{I}_{x}^{\mu a}}+\frac{\delta\Gamma}{\delta\omega_{x}^{a}}\frac{\delta\Gamma}{\delta\bar{I}_{x}^{a}}+\frac{\delta\Gamma}{\delta\bar{\omega}_{x}^{a}}\frac{\delta\Gamma}{\delta K_{x}^{a}}\Big)=0. \quad (25)$$

After differentiating (24) and (25) with respect to S, ω and $\bar{\omega}$ so as to leave zero ghost number, the equations translate into identities amongst the strongly connected Green functions, as follows. On the ω equation, apply

$$\frac{\delta}{\delta\omega^{b}},\qquad \frac{\delta^{2}}{\delta\omega^{b}\,\delta A^{\nu c}},\qquad \frac{\delta^{3}}{\delta\omega^{b}\,\delta A^{\nu c}\,\delta A^{\lambda d}},\qquad \frac{\delta^{3}}{\delta\omega^{b}\,\delta\bar{\omega}^{c}\,\delta\omega^{d}}.$$

On the BRS identity, apply

$$\frac{\delta^{2}}{\delta\omega^{b}\,\delta A^{\nu c}},\qquad \frac{\delta^{3}}{\delta\omega^{b}\,\delta A^{\nu c}\,\delta A^{\lambda d}},\qquad \frac{\delta^{4}}{\delta\omega^{b}\,\delta A^{\nu c}\,\delta A^{\lambda d}\,\delta A^{\kappa e}},$$

$$\frac{\delta^{3}}{\delta\omega^{b}\,\delta\bar{\omega}^{c}\,\delta\omega^{d}},\qquad \frac{\delta^{4}}{\delta\omega^{b}\,\delta\bar{\omega}^{c}\,\delta\omega^{d}\,\delta A^{\nu e}}.$$

Thus we obtain (integration over x is implied in (27), but not over other spatial arguments):

$$\partial^\mu_x \frac{\delta^2\Gamma}{\delta \bar{I}^{\mu a}\,\delta\omega^b} = -\frac{\delta^2\Gamma}{\delta\bar{\omega}^a\,\delta\omega^b} + \frac{\xi}{2}\frac{\delta^2\Gamma}{\delta \bar{K}^a_x\,\delta\omega^b}, \tag{26a}$$

$$\partial^\mu_x \frac{\delta^3\Gamma}{\delta \bar{I}^{\mu a}_x\,\delta\omega^b\,\delta A^{\nu c}} = -\frac{\delta^3\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta A^{\nu c}} + \frac{\xi}{2}\frac{\delta^3\Gamma}{\delta \bar{K}^a_x\,\delta\omega^b\,\delta A^{\nu c}}, \tag{26b}$$

$$\partial^\mu_x \frac{\delta^4\Gamma}{\delta \bar{I}^{\mu a}_x\,\delta\omega^b\,\delta A^{\nu c}\,\delta A^{\lambda d}} = -\frac{\delta^4\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta A^{\nu c}\,\delta A^{\lambda d}} + \frac{\xi}{2}\frac{\delta^4\Gamma}{\delta \bar{K}^a\,\delta\omega^b\,\delta A^{\nu c}\,\delta A^{\lambda d}}, \tag{26c}$$

$$\partial^\mu_x \frac{\delta^4\Gamma}{\delta \bar{I}^{\mu a}_x\,\delta\omega^b\,\delta\bar{\omega}^c\,\delta\omega^d} = -\frac{\delta^4\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta\bar{\omega}^c\,\delta\omega^d} + \frac{\xi}{2}\frac{\delta^4\Gamma}{\delta \bar{K}^a_x\,\delta\omega^b\,\delta\bar{\omega}^c\,\delta\omega^d}; \tag{26d}$$

$$\frac{\delta^2\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}}\cdot\frac{\delta^2\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b} + \frac{\delta^2\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b}\cdot\frac{\delta^2\Gamma}{\delta K^a_x\,\delta A^{\nu c}} = 0, \tag{27a}$$

$$\begin{aligned}
&\frac{\delta^3\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}\,\delta A^{\lambda d}}\cdot\frac{\delta^2\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b}\;\frac{\delta^2\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b}\cdot\frac{\delta^3\Gamma}{\delta K^a_x\,\delta A^{\nu c}\,\delta A^{\lambda d}}\\
&\quad+\left(\frac{\delta^2\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}}\cdot\frac{\delta^3\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b\,\delta A^{\lambda d}} + \frac{\delta^3\Gamma}{\delta\bar{\omega}^a\,\delta\omega^b\,\delta A^{\nu c}}\cdot\frac{\delta^2\Gamma}{\delta K^a_x\,\delta A^{\lambda d}} + (cd)\right) = 0,
\end{aligned} \tag{27b}$$

$$\begin{aligned}
&\frac{\delta^4\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}\,\delta A^{\lambda d}\,\delta A^{\kappa e}}\cdot\frac{\delta^2\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b} + \frac{\delta^2\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b}\cdot\frac{\delta^4\Gamma}{\delta K^a_x\,\delta A^{\nu c}\,\delta A^{\lambda d}\,\delta A^{\kappa e}}\\
&\quad+\left(\frac{\delta^2\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}}\cdot\frac{\delta^4\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^c\,\delta A^{\lambda d}\,\delta A^{\kappa e}} + \frac{\delta^3\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu c}\,\delta A^{\lambda d}}\cdot\frac{\delta^3\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b\,\delta A^{\kappa e}}\right.\\
&\quad+\frac{\delta^3\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta A^{\kappa e}}\cdot\frac{\delta^3\Gamma}{\delta K^a_x\,\delta A^{\nu c}\,\delta A^{\lambda d}}\\
&\quad\left.+\frac{\delta^4\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta A^{\lambda d}\,\delta A^{\kappa e}}\cdot\frac{\delta^2\Gamma}{\delta K^a_x\,\delta A^{\nu c}} + (cde)\right) = 0,
\end{aligned} \tag{27c}$$

$$\begin{aligned}
&\frac{\delta^2\Gamma}{\delta\bar{\omega}^c\,\delta\omega^a_x}\cdot\frac{\delta^3\Gamma}{\delta\bar{I}^a_x\,\delta\omega^b\,\delta\omega^d} + \left(\frac{\delta^3\Gamma}{\delta A^{\mu a}_x\,\delta\omega^b\,\delta\bar{\omega}^c}\cdot\frac{\delta^2\Gamma}{\delta\bar{I}^a_{\mu x}\,\delta\omega^d} - (bd)\right)\\
&\quad+\left(\frac{\delta^2\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b}\cdot\frac{\delta^3\Gamma}{\delta K^a_x\,\delta\bar{\omega}^c\,\delta\omega^d} - (bd)\right) = 0,
\end{aligned} \tag{27d}$$

$$\begin{aligned}
&\frac{\delta^2\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu e}}\cdot\frac{\delta^4\Gamma}{\delta\bar{I}^{\mu a}_x\,\delta\omega^b\,\delta\bar{\omega}^c\,\delta\omega^d} + \left(\frac{\delta^4\Gamma}{\delta A^{\mu a}_x\,\delta A^{\nu e}\,\delta\bar{\omega}^c\,\delta\omega^d}\cdot\frac{\delta^2\Gamma}{\delta\bar{I}^a_{\mu x}\,\delta\omega^b} - (bd)\right)\\
&\quad+\frac{\delta^3\Gamma}{\delta A^{\nu e}\,\delta\bar{\omega}^c\,\delta\omega^a_x}\cdot\frac{\delta^3\Gamma}{\delta\bar{I}^a_x\,\delta\omega^b\,\delta\omega^d} + \frac{\delta^4\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b\,\delta\bar{\omega}^c\,\delta\omega^d}\cdot\frac{\delta^2\Gamma}{\delta K^a_x\,\delta A^{\nu e}}\\
&\quad+\left(\frac{\delta^3\Gamma}{\delta A^{\mu a}_x\,\delta\bar{\omega}^c\,\delta\omega^d}\cdot\frac{\delta^3\Gamma}{\delta\bar{I}^a_{\mu x}\,\delta\omega^b\,\delta A^{\nu e}} - (bd)\right)\\
&\quad+\left(\frac{\delta^2\Gamma}{\delta\bar{\omega}^a_x\,\delta\omega^b}\cdot\frac{\delta^4\Gamma}{\delta K^a_x\,\delta A^{\nu e}\,\delta\bar{\omega}^c\,\delta\omega^d} - (bd)\right) + \frac{\delta^2\Gamma}{\delta\omega^a_x\,\delta\bar{\omega}^c}\cdot\frac{\delta^4\Gamma}{\delta\bar{I}^a_x\,\delta\omega^b\,\delta\omega^d\,\delta A^{\nu e}}
\end{aligned}$$

$$+\left(\frac{\delta^3\Gamma}{\delta\bar\omega_x^a\,\delta\omega^b\,\delta A^{\nu e}}\cdot\frac{\delta^3\Gamma}{\delta K_x^a\,\delta\bar\omega^c\,\delta\omega^d}-(bd)\right)=0. \tag{27e}$$

These identities can be verified at tree level using the Feynman rules (figure 1: those for the sources follow trivially from (18)). For example, (27a) becomes (figure 2)

$$(-\eta_{\mu\nu}k^2+k_\mu k_\nu(1-\xi^{-1})(-\mathrm{i}k^\mu\delta^{bc})+(-\mathrm{i}\xi^{-1}k_\nu)(k^2\delta^{bc})=0.$$

At one-loop order the identity is given in figure 3, and is verified by an explicit calculation.

Figure 2. A tree-level BRS identity.

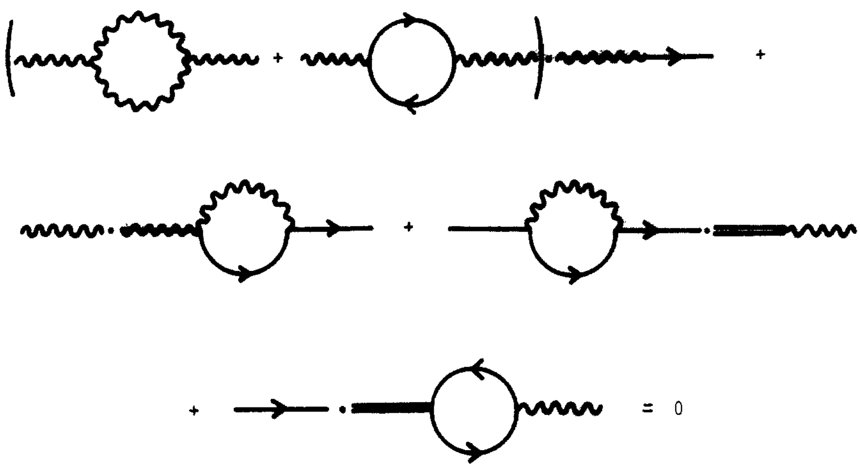

Figure 3. BRS identity at one-loop level.

Renormalisation is effected by introducing appropriate $(Z-1)\mathscr{L}$-type counterterms in order to cancel the quantum correction infinities of the effective action. The superficially divergent graphs correspond to the original terms in but *may* also include†

$$\frac{\delta^4\Gamma}{\delta A\,\delta A\,\delta\bar\omega\,\delta\omega}\ \text{(as in scalar electrodynamics)},\qquad \text{and}\ \frac{\delta^4\Gamma}{\delta\bar K\,\delta\omega\,\delta A\,\delta A}.$$

These functions do not arise at tree level but enter the general identities (26) and (27). Because they are not included in $\mathscr{L}$, we know that any potential divergences must cancel by symmetry requirements. An explicit check at one-loop level (taking zero external momentum for simplicity and ignoring infrared problems there) bears this out.

In a similar vein, it is the Sp(2) symmetry which dictates the renormalisation of the three-point ghost vertex $\delta^3\Gamma/\delta A\,\delta\bar\omega\,\delta\omega$. In spinor notation, the only possible coupling is

$$A^\mu\cdot\omega^\alpha\times\partial_\mu\omega_\alpha\propto A^\mu\cdot\bar\omega\times\overleftrightarrow{\partial}_\mu\omega,$$

† Note that $\delta^3\Gamma/\delta K\,\delta A\,\delta A$ vanishes identically.

since $\partial^\mu A_\mu \cdot \omega^\alpha \times \omega_\alpha \equiv 0$. The only slight complications occur in the source terms: additional contributions to $\delta^2\Gamma/\delta\bar{K}\,\delta\omega$ and $\delta^2\Gamma/\delta\bar{K}\,\delta A\,\delta\omega$ are handled by extra counterterms. Finally, then, the renormalised Lagrangian reads

$$\begin{aligned}\mathscr{L} = -\tfrac{1}{4}ZF^a_{\mu\nu}F^{\mu\nu a} &- (1/2\xi)Z''(\partial\cdot A^a)^2 - \tilde{Z}\partial^\mu\bar{\omega}^a\partial_\mu\omega_a - \tfrac{1}{2}e\tilde{Z}_e f^{abc}A^{\mu a}\bar{\omega}^b\overleftrightarrow{\partial}_\mu\omega^c \\ &+ \tfrac{1}{8}e^2\xi\tilde{Z}_4(f^{abc}\bar{\omega}^b\omega^c)^2 + \tfrac{1}{4}e^4Z_4(f^{abc}A^b_\mu A^c_\nu)^2 - j^{\mu a}A^a_\mu - \bar{J}^a\omega^a - \bar{\omega}^aJ^a \\ &+ \tfrac{1}{2}e\bar{\bar{Z}}_e f^{abc}\bar{\bar{I}}^a\omega^b\omega^c - \bar{I}^{\mu a}(\bar{Z}\partial_\mu\omega^a + e\bar{Z}_e f^{abc}A^b_\mu\omega^c) \\ &- K^a(\hat{Z}\partial\cdot A^a/\xi - \tfrac{1}{2}ef^{abc}\hat{Z}_e\bar{\omega}^b\omega^c) \\ &- ef^{abc}\bar{K}^a(\hat{\tilde{Z}}_e\partial\cdot A^b/\xi + \tfrac{1}{2}e\hat{\tilde{Z}}_4 f^{bde}\bar{\omega}^d\omega^e)\omega^c \\ &- e(\delta\hat{\tilde{Z}}_e/\xi)f^{abc}\bar{K}^aA^{\mu b}\partial_\mu\omega^c - \delta\tilde{Z}\partial_\mu\bar{K}^a\partial^\mu\omega^a,\end{aligned}$$

with

$$F^a_{\mu\nu} \equiv \partial_\mu A^a - \partial_\nu A^a_\mu + eZ_eZ^{-1}f^{abc}A^b_\mu A^c_\nu. \tag{28}$$

By equating infinite parts†, the equations of motion and BRS identities translate into relationships amongst the different Z's. For example, (27a) yields

$$(p_\nu p^2/\xi)[(1-Z'')+(1-\bar{Z})-(1-\tilde{Z})-(1-\hat{Z})]=0.$$

The complete set of relations derivable from (26) and (27) in this manner is

$$Z/Z_e = \tilde{Z}/\tilde{Z}_e, \qquad Z_4 = 0, \qquad \tilde{Z}_4 = \tilde{Z}_e^2/Z'' \tag{29}$$

for field renormalisations, and

$$\begin{aligned}&\hat{Z} = Z''\bar{Z}/\tilde{Z}, \qquad \bar{Z}/\bar{Z}_e = Z/Z_e, \qquad \hat{\tilde{Z}}_4 = \tilde{Z}_4, \\ &\bar{Z} = \tilde{Z} + \delta\tilde{Z}, \qquad \bar{Z}_e = \hat{Z}_e = \bar{\bar{Z}}_e = \tilde{Z}_e + \tfrac{1}{2}\delta\hat{\tilde{Z}}_e = \tfrac{1}{2}(\tilde{Z}_e + \hat{\tilde{Z}}_e),\end{aligned} \tag{30}$$

for source renormalisations. These identities may be verified to one-loop order using their calculated values given in table 1.

Table 1. Renormalisation constants in one-loop order. C is the adjoint representation Casimir invariant and $L = (e^2/16\pi^2)\log(\Lambda^2/\mu^2)$.

Z	$1+\frac{13-3\xi}{6}CL$	$\bar{Z}$	$1+\frac{3}{4}\xi CL$
		$\bar{Z}_e$	$1-\frac{1}{4}\xi CL$
Z_e	$1+\frac{17-9\xi}{12}CL$	$\hat{Z}$	1
		$\hat{Z}_e$	$1-\frac{1}{4}\xi CL$
$\tilde{Z}$	$1+\frac{1}{4}(3-\xi)CL$	$\hat{\tilde{Z}}_e$	1
$\tilde{Z}_e$	$1-\frac{1}{2}\xi CL$	$\hat{\tilde{Z}}_4$	$1-\frac{3}{4}\xi CL$
Z''	$1-\frac{1}{4}\xi CL$	$\bar{\bar{Z}}_e$	$1-\frac{1}{4}\xi CL$
Z_4	0	$\delta\tilde{Z}$	$\frac{1}{4}\xi CL$
$\tilde{Z}_4$	$1-\frac{3}{4}\xi CL$	$\delta\hat{\tilde{Z}}_e$	$\frac{1}{2}\xi CL$

† Strictly speaking, one could introduce additional sources at the bare level for the extra terms involving $\delta\tilde{Z}$ and $\delta\hat{\tilde{Z}}_e$. However, the results are unaffected at the one-loop level. Further, we have omitted a quantum-loop induced δZKK counterterm, which does not contribute to the equation of motion.

Equations (29) and (30) mean that infinities can be precisely associated with *multiplicative* renormalisations of the fields and coupling in the original bare Lagrangian (subscript o) wherever they appear:

$$A_o = Z^{1/2} A, \qquad \omega_o = \tilde{Z}^{1/2}\omega, \qquad e_o = eZ_e/Z^{3/2}, \qquad \xi_o = \xi Z/Z''. \tag{31}$$

In particular, (29) includes the Slavnov–Taylor identity (Taylor 1971, Slavnov 1972) $Z/Z_e = \tilde{Z}/\tilde{Z}_e$. By the same token the sources, and indeed the BRS transformations themselves, undergo multiplicative renormalisation.

5. Super-Lorentz transformations

The action (11) plus (12) is formally invariant under the super-Lorentz transformations

$$\delta\Phi_u(x, \theta) = \Lambda_u^{\ v}\Phi_v(\Lambda^{-1}(x, \theta)) - \Phi_v(x, \theta), \tag{32}$$

where $\Lambda(x, \theta)$ is given by (4). However, these transformations do *not* respect the condition (8) defining the restricted class of gauge potentials. In contrast to the case of supertranslations, the variations implied by (32) are incompatible with the parametrisation of the components in (13) in terms of just *four* fields $A_\mu(x)$, $\omega_\alpha(x)$ and $B(x)$.

A set of transformations can still be obtained from (32) by extracting the variations of $A_\mu(x)$, $\omega_\alpha(x)$ and $B(x)$ from the lowest-order components in (13), as if the remaining variations were consistent. In the case of $\delta A_\mu(x)$, (32) and (13) conspire to give a ghost-dependent gauge transformation given by

$$\delta A_\mu = D_\mu(\bar{\lambda}_\nu x^\nu \omega) - D_\mu(\lambda_\nu x^\nu \bar{\omega}) \tag{33}$$

in terms of the infinitesimal spinor components $\bar{\lambda}_\nu$ and λ_ν. Comparing (33) with (17*a*), a formal similarity with an x-dependent BRS transformation is seen. The complete set of transformations, in terms of the appropriately scaled variables, is

$$\begin{aligned} &\delta A = D_\mu(\bar{\lambda}_\nu x^\nu \omega), \qquad \delta\omega = \bar{\lambda}_\nu x^\nu \omega \times \omega, \qquad \delta\bar{\omega} = (2/\xi)\bar{\lambda}^\mu A_\mu - \bar{\lambda}_\nu x^\nu B_+, \\ &\qquad \delta B_+ = -(2/\xi)\bar{\lambda}^\mu D_\mu \omega, \qquad \delta B_- = -\xi\bar{\lambda}^\mu \partial_\mu \omega + \bar{\lambda}_\nu x^\nu B_- \times \omega, \end{aligned} \tag{34}$$

plus a similar set in terms of λ_ν, derived by Hermitian conjugation (cf (17)).

The basic set (34) further imply

$$\delta D_\mu \omega = \tfrac{1}{2} e \bar{\lambda}_\mu \omega \times \omega, \qquad \delta\omega \times \omega = 0, \qquad \delta B_- \times \omega = -(1/\xi)\bar{\lambda}^\mu \partial_\mu(\omega \times \omega),$$

from which

$$\delta\mathscr{L} = -(2/\xi)\bar{\lambda}^\mu F^a_{\mu\nu}\partial^\nu \omega^a. \tag{35}$$

Furthermore, it can be verified that the variations (34) are consistent with the equations of motion (19), in the absence of sources. Thus the auxiliary field B can be eliminated, as in the supertranslation case. The identity for the generating functional resulting from (34) and (35) is

$$\begin{aligned} \int \mathrm{d}x \Big[& j_\mu \cdot \mathrm{i}\frac{\delta}{\delta \bar{J}} - \bar{I}_\mu \cdot \mathrm{i}\frac{\delta}{\delta \bar{I}} + x_\mu\Big(j^\nu \cdot \mathrm{i}\frac{\delta}{\delta \bar{I}^\nu} + J \cdot \mathrm{i}\frac{\delta}{\delta K} - \bar{J} \cdot \mathrm{i}\frac{\delta}{\delta \bar{I}}\Big) \\ & -\frac{2}{\xi}\Big(J \cdot \mathrm{i}\frac{\delta}{\delta j^\mu} - F_{\mu\nu}\Big(\mathrm{i}\frac{\delta}{\delta j}\Big) \cdot \partial^\nu \mathrm{i}\frac{\delta}{\delta \bar{J}} + K \cdot \mathrm{i}\frac{\delta}{\delta \bar{I}^\mu} + \bar{K} \cdot \mathrm{i}\frac{\delta}{\delta \bar{I}}\Big)\Big] Z = 0. \end{aligned}$$

Some simplification is possible with the use of the equation of motion, (19). In fact, in the Landau gauge $\xi \to 0$, (34) and (35) are equivalent to (19), in the absence of composite sources. Passing to the effective action, one finds in general that

$$\begin{aligned}\int \mathrm{d}x \Bigg[A_\mu \cdot \frac{\delta\Gamma}{\delta\bar{K}} - \omega \cdot \frac{\delta\Gamma}{\delta A^\mu} + \bar{I}^\mu \cdot \frac{\delta\Gamma}{\delta\bar{\bar{I}}} - \mathrm{i}\Delta_{\mu\lambda} \cdot \frac{\delta^2\Gamma}{\delta A^\lambda \delta\bar{K}} \\ + x_\mu \left(\frac{\delta\Gamma}{\delta A^\lambda} \cdot \frac{\delta\Gamma}{\delta\bar{I}_\lambda} + \frac{\delta\Gamma}{\delta\omega} \cdot \frac{\delta\Gamma}{\delta\bar{\bar{I}}} + \frac{\delta\Gamma}{\delta\bar{\omega}} \cdot \frac{\delta\Gamma}{\delta K} \right) \\ + \frac{1}{\xi} \bigg(2\partial^\lambda A_\lambda \cdot \frac{\delta\Gamma}{\delta\bar{I}^\mu} - \partial_\mu\omega \cdot K + K \cdot \frac{\delta\Gamma}{\delta\bar{I}^\mu} - e\bar{K} \cdot A_\mu \times \frac{\delta\Gamma}{\delta\bar{\bar{I}}} \\ + 2\bar{K} \cdot \partial_\mu \frac{\delta\Gamma}{\delta\bar{\bar{I}}} - 2\mathrm{i}\partial^\kappa \Delta_{\kappa\lambda} \frac{\delta^2\Gamma}{\delta A^\lambda \delta\bar{I}^\mu} + \mathrm{i}e\bar{K} \cdot \Delta_{\mu\lambda} \frac{\delta^2\Gamma}{\delta A^\lambda \times \delta\bar{\bar{I}}} \bigg) \Bigg] = 0 \qquad (36)\end{aligned}$$

where

$$\Delta_{\mu\lambda} = \delta^2 W / \delta_j^\mu \delta j^\lambda,$$

and an additional integral over the spatial coordinate is understood in the appropriate $\Delta_{\mu\lambda}$ terms.

6. Conclusions

We have seen above that an alternative formulation of gauge fixing in pure Yang–Mills theory, based upon inhomogeneous OSp(4/2) space–time supersymmetry, leads naturally to the model Lagrangian (15). Here a covariant gauge-fixing term for the vector potential is accompanied by non-standard ghost couplings, including a four-point coupling (cf Das 1980). As a result of additional symmetry between ghost and anti-ghost fields, the Lagrangian is formally real if these are complex conjugates (in contrast to the usual case). The model is renormalisable in standard fashion, and the BRS transformations lead to the Slavnov–Taylor identity for the renormalisation of the transverse part of the vector propagator. The renormalisation constants are given to one-loop order in table 1.

Bonora *et al* (1980) have justified their superfield formalism in terms of a fibre bundle construction. In this and other geometrical approaches (Thierry-Mieg and Ne'eman 1979, Quirós *et al* 1980), no discussion has been given of a possible enlarged space–time supersymmetry. With this at hand, however, the model can be regarded as being obtained by dimensional reduction from the six-dimensional theory. Indeed, the condition (9) can be interpreted as the usual one of 'triviality in higher dimensions'. Naturally this breaks the fullOSp(4/2) supergroup, but respects supertranslations, and Sp(2) transformations.

The present formulation of gauge fixing and ghosts can be applied straightforwardly to other models. For an antisymmetrical rank-two tensor gauge field, the appropriate OSp(4/2) representation is the rank-two graded anti-symmetrical tensor representation (the 17-dimensional adjoint representation, as in (5)). This contains six gauge fields $A_{[\mu\nu]}$, eight ghosts $A_{\mu\alpha}$, and three scalar fields $A_{(\alpha\beta)}$, with graded dimensions $6-8+3=1$, as is appropriate for a scalar field (Siegel 1980, Marchetti and Tonin 1981). For gravity, the irreducible rank-two traceless graded-symmetrical tensor representation of OSp(4/2) is 18-dimensional (note that $17+(18+1)=36$), in accord with the usual assignment (Delbourgo and Medrano 1976) of two vector ghosts $g_{\mu\alpha}$ accompanying the gravition field $g_{(\mu\nu)}$, and graded dimension $10-8=2$. Including the OSp(4/2) trace, the reducible 19-dimensional representation may correspond to a scalar-tensor theory (see also Namazie and Storey 1979).

More generally, one can consider 'extended' ghost supersymmetries OSp$(4/G)$ in G dimensions, where G is even. For example, the rank-three graded-antisymmetrical tensor representation of OSp$(4/G)$ (including the gauge field $A_{[\lambda\mu\nu]}$) has graded dimension $\frac{1}{6}(4-G)(3-G)(2-G)$, suggesting that for $G=2$ or $G=4$, the theory has zero physical degrees of freedom. Similarly, consider the rank-three gauge field of mixed symmetry proposed as a representation of a massless spin-1 field (Curtright 1980). The appropriate OSp$(4/G)$ representation would appear to be the reducible rank-three tensor, of graded mixed symmetry type (with non-zero graded trace). The graded dimension (the same as that in SU$(4/G)$) is $\frac{1}{3}(4-G)(5-G)(3-G)$ (cf Bars and Balantekin 1981). The count of degrees of freedom is thus correct for $G=2$, while for $G=4$ the theory appears to be null. In the former case, the ghost assignments, from O(4) × Sp(2) reduction (the same as the SU(4) × SU(2) case: Dondi and Jarvis (1981) are: 20 gauge fields $A_{[\mu\nu]\lambda}$; 32 ghost fields $A_{\mu\nu\alpha}$; 16 gauge fields $A_{\mu\alpha\beta}$, and 2 ghost fields $A_{[\alpha\beta]\gamma}$. As a final example, the graded dimension of the irreducible totally graded-symmetrical graded-traceless rank-three tensor representation of OSp$(4/G)$ is $\frac{1}{6}(8-G)(3-G)(4-G)$, suggesting that the structure of a symmetric rank-three tensor gauge theory is for $G=2$: 20 gauge fields $A_{(\lambda\mu\nu)}$, 20 ghost fields $A_{(\lambda\mu)\alpha}$, 4 gauge fields $A_{\lambda[\alpha\beta]}$, supplemented by $4-2=2$ trace conditions, leaving two degrees of freedom appropriate to a massless (spin-3) gauge field. Because it is so concise and simple, we would recommend this method of counting the families of ghosts in preference to others.

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Note added in proof. After this work was completed and submitted for publication we received a preprint (Columbia University No 196) by Baulieu and Thierry-Mieg (*Nucl. Phys.* to be published) which is rather similar in content. Our Lagrangian (15) corresponds to their Hamiltonian case. However, they do not develop the consequent Sp(2) symmetry nor the OSp(4/2) supersymmetric enlargement.

References

Abers E S and Lee B W 1973 *Phys. Rep.* **9**
Bars I and Balantekin A B 1981 *J. Math. Phys.* **22** 1149
Becchi C, Rouet A and Stora A 1975 *Commun. Math. Phys.* **42** 127
—— 1976 *Ann. Phys., NY* **98** 287
Bonora L, Pasti P and Tonin M 1980 *Padua Preprint IFPD* 24/80
Bonora L and Tonin M 1981 *Phys. Lett.* **98B** 48
Curci G and Ferrari R 1976 *Phys. Lett.* **63B** 91
Curtright T 1980 *Chicago Preprint EFI* 80/04
Das A 1980 *Maryland Preprint* 81–089
Delbourgo R and Medrano, R M 1976 *Nucl. Phys.* B **110** 467
De Witt B S 1965 *Dynamical Theory of Groups and Fields* (New York: Gordon and Breach)

Dondi P H and Jarvis P D 1979 *Phys. Lett.* **84B** 75; *Erratum* **87B** 403
——1980 *Z. Phys.* C **4** 201
——1981 *J. Phys. A: Math. Gen.* **14** 547
Faddeev L D and Popov V N 1967 *Phys. Lett.* **25B** 29
Fayet P and Ferrara S 1977 *Phys. Rep.* **32** 69
Ferrara S, Piguet O and Schweda M 1977 *Nucl. Phys.* B **119** 493
Feynman R P 1963 *Acta Phys. Polon.* **26** 697
Kugo T and Ojima I 1979 *Prog. Theor. Phys. Suppl.* **66**
Fujikawa K 1978 *Prog. Theor. Phys.* **59** 2045
Marchetti P A and Tonin M 1981 *Padua Preprint IFPD* 32/81
Namazie M A and Storey D 1979 *Nucl. Phys.* B **157** 170
Ojima I 1980 *Prog. Theor. Phys.* **64** 625
Parker M 1980 *J. Math. Phys.* **21** 689
Quirós M, de Urríes F J, Hoyos J, Mazón M L and Rodriguez E 1980 *Madrid Preprint IEM*
Salam A and Strathdee J 1974 *Nucl. Phys.* B **76** 477
Siegel W 1980 *Phys. Lett.* **93B** 170
Slavnov A A 1972 *Sov. Phys.-JETP* **10** 99
Taylor J C 1971 *Nucl. Phys.* B **33** 436
Thierry-Mieg J and Ne'eman Y 1979 *Ann. Phys., NY* **123** 247